Inverse Variation in Pressure Data

Let's take a sample of air in a closed container, and keep it at room temperature. If you change the volume of the container, what will happen to the air pressure inside? You can feel this by squeezing a small balloon in your hand. As the balloon gets smaller, you have to push harder. That is, as the volume decreases, the pressure increases. Two quantities that change in this way could be *inversely* related. If pressure and volume are inversely related, even if both quantities change, then their product stays the same.

Suppose that x and y represent the quantities that are inversely related. Then

$$xy = k, \quad \text{or} \quad y = \frac{k}{x}$$

where k is a constant in both equations. Maybe you can think of some other quantities that also behave this way. For air and other gases, this relation has a name: *Boyle's law.*

In this activity, you will use a pressure sensor to investigate the relationship between pressure and volume for air contained within a closed syringe.
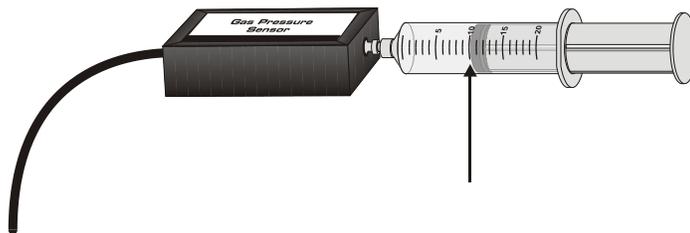
OBJECTIVES

- Record pressure versus volume data for a sample of air.
- Fit an inverse function model to the data.
- Re-plot the data using linearization.

MATERIALS

TI-Nspire handheld
EasyLink interface
Vernier Pressure Sensor with included syringe

PROCEDURE

1. Prepare the Pressure Sensor and syringe for data collection.

    a. Connect the Pressure Sensor to the EasyLink interface. Connect the EasyLink to the TI-Nspire handheld.

b. With the syringe disconnected from the Pressure Sensor, move the piston of the syringe until the leading edge of the inside black ring is positioned at the 10.0 mL mark.
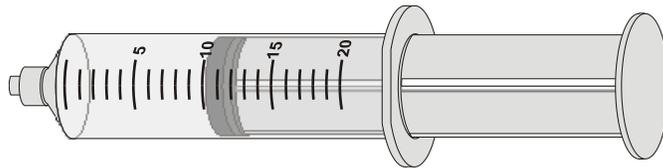
c. Attach the syringe to the Pressure Sensor. The air in the syringe/Pressure Sensor system is now sealed, so that as you move the piston, the air volume and pressure both change.

2. To obtain the best data possible, you will need to correct the volume readings from the syringe. Look at the syringe; its scale reports its own internal volume. However, that volume is not the total volume of trapped air in your system since there is a little bit of space in the tubing inside the pressure sensor.

   To account for the extra volume in the system, you will need to add a small and constant volume to your syringe readings. The correction is 0.8 mL. For example, with the 0.8 mL correction and a 5.0 mL syringe volume, the total volume would be 5.8 mL. *It is the total volume that you will need for the analysis.*

3. Set up DataQuest for data collection.

   a. Choose New Experiment from the Experiment menu.
   b. Choose Collection Mode ► Events with Entry from the Experiment menu.
   c. Enter **Volume** as the Name and **mL** as the Units. Select OK.

4. You are now ready to collect pressure and volume data. It is best for one person to handle the gas syringe and for another to operate DataQuest.

   a. Start data collection.
   b. Move the piston so the front edge of the inside black ring is positioned at the 5 mL line on the syringe. Hold the piston firmly in this position until the pressure value displayed on the screen stabilizes.
   c. Click the Keep button. (The person holding the syringe can relax after the Keep button has been clicked.) Enter **5.8**, the total gas volume (in mL). Remember, you are adding the constant volume to the apparent volume of the syringe. Select OK to store this data pair.
   d. To collect another data pair, move the piston to 7.0 mL. When the pressure reading stabilizes, click the Keep button, enter the total volume, remembering to add the correction to the syringe volume, and then select OK.
   e. After the second point is collected, select Autoscale Now from the ⌁ Graph menu.
   f. Continue collecting data using volumes of 10.0, 12.0, 15.0, 17.0, and 20.0 mL. You will not need to repeat the auto-scaling step.
   g. Stop data collection when you have finished collecting data. A graph of pressure versus volume will be shown.

DATA TABLE

Total Volume (*x* values)	Pressure (*y* values)	Product (*x**y)

Volume (mL) (*x* values)	Pressure (kPa) (*y* values)
2.5	
17.8	
520	
0.0012	

ANALYSIS

1. Click any data point and use ▶ and ◀ to examine the data pairs on the graph. As you move across the graph, the volume (*x*) and pressure (*y*) values of each data point are displayed. Record the pressure and volume data values in your data table.

2. To test Boyle's law you can plot the inverse function $y = k/x$ with your data. You will see this equation with the data, and can then adjust the value for *k* to improve the fit.

   a. Insert a Graphs page.
   b. Insert the Sensor Console in order to input the graph from DataQuest. Verify that your data appears and then close the Sensor Console.
   c. Choose Zoom – Data from the ⊞ Window/Zoom menu to view all of your data.
   d. Choose Function from the ⊹ Graph Type menu.
   e. Enter your model equation into the Entry Line:
   $$f1(x) = k/x$$
   f. Choose Insert Slider from the ▸ Actions menu and place the slider box onto a clear portion of the graph.
   g. Change the variable in the slider box to *k*.
   h. Change the range of the slider. To do this, click the maximum value on the slider range and enter 1500, and enter 500 for the minimum value.
   i. Adjust the slider to change value of *k* until the model fits the data and then answer Analysis Question 1.

3. Find the products of the data coordinates and record them in the third column of the Data Table. Then, answer Analysis Question 2.

4. The modeling equation you have determined in this activity can be used to predict syringe pressure values for given volumes. The second data table lists a number of volumes. Fill in the corresponding pressures and then answer Analysis Questions 3 and 4.

   Another way to see if two quantities are inversely related is to plot the one quantity versus the inverse of the other. For this pressure activity, that would be pressure versus the inverse of the volume. If the new plot shows a line of proportionality, then the original data is inversely related. You can see this by noting the simple rearrangement of the equation $y = k/x$ as $y = k \times (1/x)$. Written this way, you can see that y is proportional to the quantity $(1/x)$. The process of graphing calculated quantities to obtain a straight-line graph is sometimes called *linearization*.

   To see if your data are inversely related using this test, you need to use a new column containing the inverse of the volume data. DataQuest can be set up to calculate the inverse of the volume data.

   a. Return to your DataQuest page.
   b. Choose New Calculated Column from the Data menu.
   c. Enter **Inverse Vol** as the Name, **1/V** as the Short Name, and **1/mL** as the Units.
   d. Enter **1/Volume** as the Expression. **Note**: The term "Volume" must exactly match the name of the column. If you are unsure how it was entered, the available column names can be found below the Expression entry box.
   e. Select OK.
   f. Click the Graph View tab. Plot the modified graph by choosing Select X-axis Columns ► Inverse Vol from the Graph menu.
   g. Choose Curve Fit ► Proportional from the Analyze menu

   Answer Analysis Question 5.

ANALYSIS QUESTIONS

1. Use the value for k to record the model equation that gives the best fit to the data.

2. Notice that the values in the third column of the first data table are closely related to the value of k you found. Explain why this is so.

3. Could the volume ever be zero? Why or why not? What would be the corresponding pressure?

4. Complete the following statement:
   As the volume of a gas sample decreases, its pressure _____.

5. Based on your graph of pressure versus inverse volume, are pressure and volume inversely proportional?

EXTENSIONS

1. An alternate way to find a model for the data you collected involves a process known as *regression analysis*. DataQuest will select the best parameters for the power function $y = ax^b$.

   Note: Before performing this extension, remove the proportional curve fit by choosing Remove Fit: Proportional from the Analyze menu. Change the graph back to pressure *vs.* volume choosing Volume for the X-axis.

   a. Next, choose Curve Fit ► Power from the Analyze menu.
   b. Select OK to view the graph.

   Does the power law model provide a good fit to your pressure *vs.* volume data? How many parameters are adjusted to achieve a good fit compared to your inverse model?

2. It is possible to deduce the internal volume of the sensor rather than using the supplied values. If you assume Boyle's law, you can determine the internal volume from the *y*-intercept value. For example, if the internal volume of the sensor is *d*, then the total volume of air in the system is $V + d$. From Boyle's law, we have $P*(V + d) = k$, which can be rearranged to $V = k*(1/P) - d$. From the form of this equation, we can see that the *y*-intercept of a *V* (*y*-axis) versus $1/P$ (*x*-axis) graph is the negative of the internal volume.

   Note that this is not the same graph setup as used in the activity. Collect a new data set, but do not correct the volumes; just enter the syringe volume directly. Create a graph of *V* versus $1/P$. Fit a straight line to the data, and from the *y*-intercept determine the internal volume of your particular sensor. For comparison to your results, the internal volume of the Vernier Gas Pressure Sensor is about 0.8 mL.